

NASA Contractor Report 187211

IN-39  
51120  
P-27

# Limit Pressure of a Circumferentially Reinforced SiC/Ti Ring

(NASA-CR-187211) LIMIT PRESSURE OF A  
CIRCUMFERENTIALLY REINFORCED SiC/Ti RING  
Final Report (Akron Univ.) 27 p CSCL 20K

N92-11387

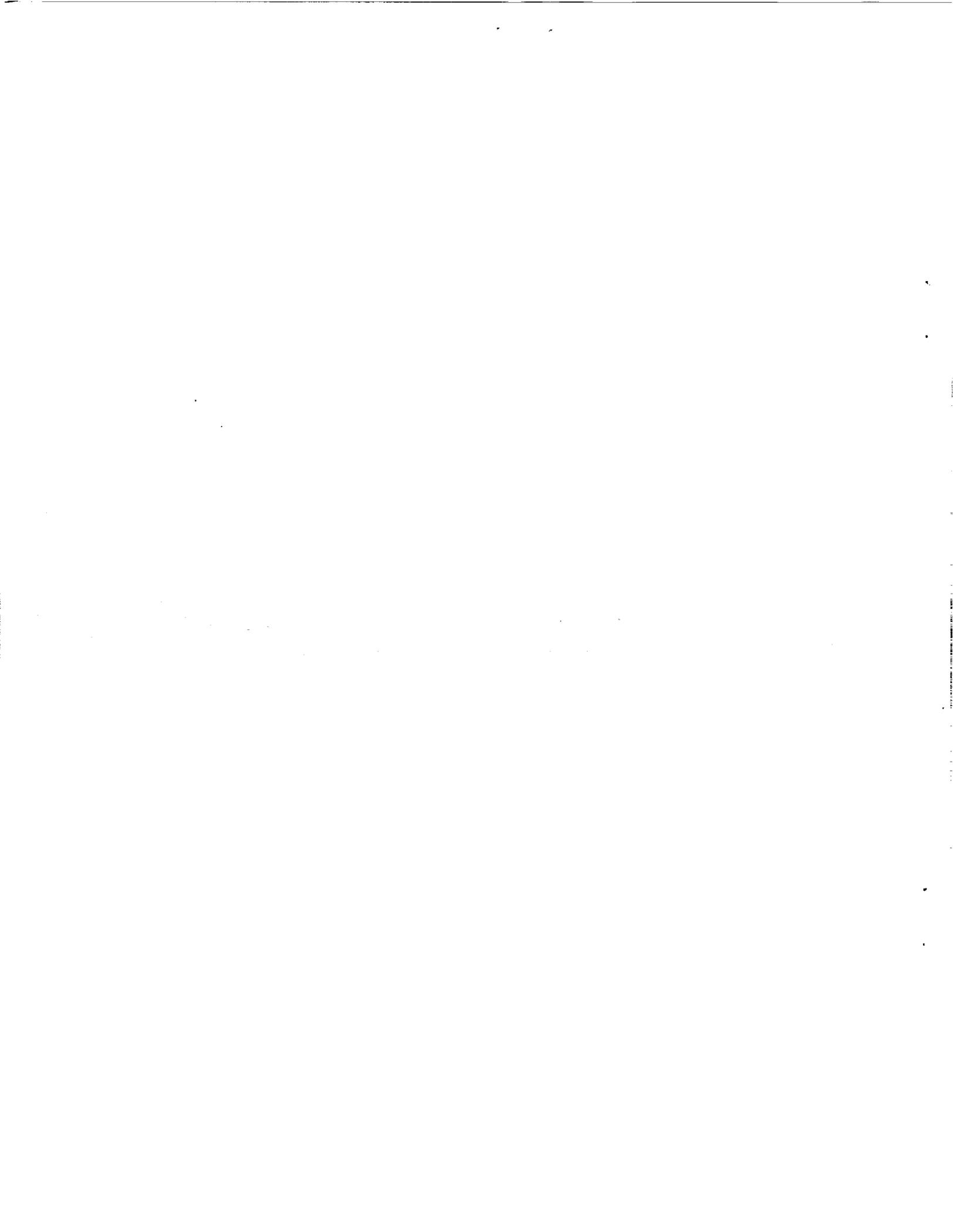
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October 1991

Prepared for  
Lewis Research Center  
Under Grant NAG3-379

**NASA**  
National Aeronautics and  
Space Administration



LIMIT PRESSURE OF A CIRCUMFERENTIALLY REINFORCED  
SiC/Ti RING

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ABSTRACT

Limit loads under plane stress and plane strain are found for a circumferentially reinforced elastic-plastic ring subjected to interior pressure. These are used as bounds on an estimate of the failure pressure of a SiC/Ti test ring that is being fabricated and tested under the co-sponsorship of NASA Lewis Research Center and Pratt and Whitney Aircraft. The ring is to serve as a benchmark against which deformation and failure analysis methods can be assessed. An anisotropic perfect plasticity idealization of the SiC/Ti ring material is made and used in the limit load calculations. An estimate of the failure pressure of the NASA/PW benchmark test ring is given.

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## INTRODUCTION

A plastic limit analysis is made for estimating the failure pressure of a SiC/Ti test ring subjected to a monotonically increasing pressure at 427 C. The circumferentially reinforced SiC/Ti ring models a critical aircraft engine component. The fabrication and testing of the ring is co-sponsored by NASA Lewis Research Center and Pratt and Whitney Aircraft as a benchmark for assessing the applicability and accuracy of structural analysis and failure prediction methods for metallic composites.

Following Robinson and Duffy (1990) and Binienda and Robinson (1990), the SiC/Ti material is considered a pseudohomogeneous, locally transversely isotropic continuum and is idealized as elastic perfectly plastic. The concepts of limit analysis of elastic-plastic continua are used under the assertion that strains large enough to cause rupture occur only as the plastic limit load  $P^l$  is reached (fig.1). This approach circumvents details such as residual stresses that affect the initial yield pressure  $P^y$  (fig.1) and the intermediate elastic-plastic response but not the limit pressure  $P^l$ . The failure pressure of the NASA/PW ring is bounded by calculating the limit pressures of the two limiting cases of plane stress and plane strain.

The objective is to provide an estimate of the failure pressure of the NASA/PW benchmark ring using approximate methods useful to the design engineer without conducting a detailed and costly structural analysis on a large computer. Limit analysis provides closed form expressions for the failure pressure, or bounds on it, valid for a range of ring geometries and material properties; such simple and flexible tools are useful to the designer, especially in the early stages of design. The limit load calculations also provide a check for verifying the correctness of a more comprehensive finite

element analysis of the NASA/PW ring.

The authors anticipate a sequel to this publication comparing the results obtained here with experimental results on the NASA/PW benchmark ring as they become available.

## LIMIT ANALYSIS

We first state the anisotropic plasticity theory used in the analysis. Following Robinson and Duffy (1990),  $d_i(x_j)$  is a unit vector field designating the local fiber direction (normal to the plane of transverse isotropy) at each material point.  $D_{ij} = d_i d_j$  is a symmetric second rank orientation tensor. The yield function  $\phi(\sigma_{ij})$  is

$$\phi(\sigma_{ij}) = J_2 - \frac{3}{4}\zeta(I^2 - K^2), \quad (1)$$

written in terms of two invariants of the deviatoric stress  $s_{ij}$  and  $D_{ij}$ , viz,

$$J_2 = \frac{1}{2} s_{ij} s_{ji} \quad (2)$$

and 
$$I = D_{ij} s_{ji} \quad (3)$$

The parameter  $0 \leq \zeta \leq 1$  designates the degree of anisotropy.  $\zeta = 0$  corresponds to full isotropy whereas  $\zeta = 1$  indicates unlimited strength in the fiber direction.  $K$  is the yield stress under transverse shear.

The flow law is

$$\dot{\epsilon}_{ij} = \lambda \Gamma_{ij} \quad (\lambda \geq 0) \quad \text{if } \phi = 0 \text{ and } \Gamma_{ij} \dot{\sigma}_{ji} = 0 \quad (4)$$

$$\dot{\epsilon}_{ij} = 0 \quad \text{if } \phi < 0 \text{ or } \phi = 0 \text{ and } \Gamma_{ij} \dot{\sigma}_{ji} < 0$$

in which 
$$\Gamma_{ij} = s_{ij} - \frac{1}{2}\zeta(I(3D_{ij} - \delta_{ij})) \quad (5)$$

Note that with  $\zeta = 0$  this theory reduces to the classical  $J_2$  (v.Mises) theory of a perfectly plastic solid.

Reduction of eqs.(1)-(5) to uniaxial stress gives

$$Y_l = K \sqrt{3/(1-\zeta)} \quad \text{and} \quad Y_t = K \sqrt{12/(4-\zeta)} \quad (6)$$

as the respective yield stresses along and transverse to the fiber direction.

In terms of cylindrical coordinates appropriate for the ring (fig.2) the nonzero stress components are  $\sigma_z, \sigma_c$  and  $\sigma_r$ . The inner and outer radii are denoted as  $a$  and  $b$ , respectively. Circumferential fiber reinforcement dictates that  $D_{22} \equiv D_c = 1$ , otherwise  $D_{ij} = 0$ .

The deviatoric stress components are

$$\begin{aligned} s_z &= \frac{1}{3}[(\sigma_z - \sigma_r) + (\sigma_z - \sigma_c)] \\ s_c &= \frac{1}{3}[(\sigma_c - \sigma_z) + (\sigma_c - \sigma_r)] \\ s_r &= \frac{1}{3}[(\sigma_r - \sigma_c) + (\sigma_r - \sigma_z)] \end{aligned} \quad (7)$$

and the components of  $\Gamma_{ij}$  are

$$\begin{aligned} \Gamma_z &= s_z + \frac{1}{2}\zeta s_c \\ \Gamma_c &= (1-\zeta) s_c \\ \Gamma_r &= s_r + \frac{1}{2}\zeta s_c \end{aligned} \quad (8)$$

In these terms, the yield function eq.(1) becomes

$$\phi = \frac{1}{2}(s_r^2 + s_z^2) - \frac{1}{4}(3\zeta - 2)s_c^2 - K^2 \quad (9)$$

and the flow law is

$$\dot{\epsilon}_p = \lambda \Gamma_p \quad \text{if } \phi = 0 \text{ and } \Gamma_p \dot{\sigma}_p = 0 \quad (10)$$

$$\dot{\epsilon}_p = 0 \quad \text{if } \phi < 0 \text{ or } \phi = 0 \text{ and } \Gamma_p \dot{\sigma}_p < 0$$

with  $p = z, c, r$

The equilibrium equation relating  $\sigma_c$  and  $\sigma_r$  is

$$r \frac{d}{dr} \sigma_r = \sigma_c - \sigma_r \quad (11)$$

and the compatibility equation in rate form is

$$\frac{d}{dr} (r \dot{\epsilon}_c) = \dot{\epsilon}_r \quad (12)$$

Generally, compatibility must be satisfied by the total strain rates. However, at the limit load where the elastic strain rates vanish, eq.(12) must be satisfied by the plastic strain rate field.

We shall first consider the plane strain problem, i.e.,  $\dot{\epsilon}_z \equiv 0$ . Using this condition in eqs.(7),(8) and (10) results in

$$\sigma_z = \left( \frac{2+\zeta}{4-\zeta} \right) \sigma_r + \left( \frac{2(1-\zeta)}{4-\zeta} \right) \sigma_c \quad (13)$$

Thus we can eliminate  $\sigma_z$  in eqs.(7) and in the yield function eq.(9) giving

$$\phi = \frac{1-\zeta}{4-\zeta} (\sigma_c - \sigma_r)^2 - K^2 = 0 \quad (14)$$

or 
$$\sigma_c - \sigma_r = \pm \sqrt{\frac{4-\zeta}{1-\zeta}} K \quad (15)$$

Fig.3 shows the yield surface, eqs.(15), in the dimensionless stress space ( $\sigma_c/2K$ ,  $\sigma_r/2K$ ). Also shown is a typical stress profile a-b for pressure loading under plane strain; evidently, the positive root in eqs.(15) is the physically meaningful one. Combining this and the equilibrium eq.(11),

yields a separable differential equation in  $\sigma_r$  and  $r$ . This equation can be integrated from  $r = a$  to  $r = b$ , making use of the boundary conditions  $\sigma_r(a) = -P^1$  and  $\sigma_r(b) = 0$ , to yield a *lower bound* on the (dimensionless) limit pressure  $P^1/2K$  for plane strain, i.e.,

$$\frac{P^1}{2K} = \frac{1}{2} \sqrt{\frac{4-\zeta}{1-\zeta}} \ln \frac{b}{a} \quad (16)$$

So far, only equilibrium, the constitutive law and the boundary conditions are shown to be satisfied (*not compatibility*), restricting eq.(16) to be a lower bound on the limit pressure.

The dimensionless equilibrium stress field at the limit pressure is

$$\frac{\sigma_r}{2K} = -\frac{1}{2} \sqrt{\frac{4-\zeta}{1-\zeta}} \ln \frac{b}{r} \quad (17)$$

$$\frac{\sigma_c}{2K} = \frac{1}{2} \sqrt{\frac{4-\zeta}{1-\zeta}} \left(1 - \ln \frac{b}{r}\right) \quad (18)$$

with  $\sigma_z/2K$  given by eq.(13). Inserting this stress field in eqs.(7),(8) and (10) results in the associated plastic strain rate field

$$\begin{aligned} \dot{\epsilon}_z &= 0 \\ \dot{\epsilon}_c &= \lambda(r) 2K \sqrt{\frac{1-\zeta}{4-\zeta}} = \frac{\lambda'}{r^2} \\ \dot{\epsilon}_r &= -\lambda(r) 2K \sqrt{\frac{1-\zeta}{4-\zeta}} = -\frac{\lambda'}{r^2} \end{aligned} \quad (19)$$

where, consistent with the perfect plasticity constitutive law,  $\lambda$  is taken as

$$\lambda(r) = \frac{1}{r^2} \quad \text{and} \quad \lambda' = 2K \sqrt{\frac{1-\zeta}{4-\zeta}} = \text{const.}$$

This strain rate field is seen to satisfy compatibility, eq.(12), thus elevating

eq.(16) to the *exact limit pressure for plane strain* under the present material idealization.

We now consider the plane stress problem in which  $\sigma_z \equiv 0$ . With this condition in eqs.(7), the yield condition eq.(1) becomes

$$\phi = \frac{1}{3} [(1-\zeta)\sigma_c^2 - (1-\zeta)\sigma_c\sigma_r + (1-\frac{\zeta}{4})\sigma_r^2] - K^2 \quad (20)$$

At yield,  $\phi = 0$ , we can solve eq.(20) for  $\sigma_c$  giving

$$\sigma_c = \frac{1}{2} [\sigma_r \pm \sqrt{\beta(4K^2 - \sigma_r^2)}] \quad ; \quad \sigma_r \leq 2K \quad (21)$$

with 
$$\beta = \frac{3}{1-\zeta}$$

The yield surface, eq.(20), is shown in the dimensionless stress space ( $\sigma_c/2K, \sigma_r/2K$ ) of fig.4 for  $\zeta = 0$  (isotropy),  $\zeta = 0.977$  ( used subsequently for the NASA/PW ring) and  $\zeta = 1$  (the limiting case of infinite strength in the fiber direction). Also indicated in fig.4 is a typical stress profile a-b for a plane stress solution under pressure loading with  $\zeta = .977$ .

As for plane strain, the positive root in eqs.(21) is the physically meaningful one. It can be used to eliminate  $\sigma_c$  in the equilibrium eq. (11), again resulting in a differential equation in  $\sigma_r$  and r only. Integrating, as earlier, over  $r = a$  to  $r = b$  and invoking the boundary conditions  $\sigma_r(a) = -P^1$  and  $\sigma_r(b) = 0$ , we obtain

$$\int_{-\frac{P^1}{2K}}^0 \frac{2 d\eta}{\sqrt{\beta(1-\eta^2)} - \eta} = \ln \frac{b}{a} \quad ; \quad \frac{P^1}{2K} \leq 1 \quad (22)$$

as a *lower bound* on the (dimensionless) limit pressure  $P^1/2K$  for plane

stress.

The dimensionless equilibrium stress field for plane stress is accordingly

$$\int_{\frac{\sigma_r}{2K}}^0 \frac{2 d\eta}{\sqrt{\beta(1-\eta^2)} - \eta} = \ln \frac{b}{r} \quad ; \quad \frac{\sigma_r}{2K} \geq -1 \quad (23)$$

with  $\sigma_c/2K$  given by eq.(21) and  $\sigma_z \equiv 0$ .

Here, it is not such a simple matter to find an associated plastic strain rate field and show compatibility. However, we recognize that the strain rate field, eqs.(19), although not associated with the equilibrium plane stress field (eqs.(23) and (21)) through the constitutive law, is nevertheless *kinematically admissible* under plane stress and thus serves for establishing an *upper bound* on the limit pressure. This upper bound is obviously the limit load corresponding to plane strain, i.e., eq.(16).

Another upper bound on the limit load for plane stress is that corresponding to a local (*out of plane*) failure at the inner radius  $r = a$  directly under the applied pressure. This upper bound is  $P^{1/2}K = 1$ , as is easily calculated using the kinematically admissible slip line field of fig.5. For this local mode to occur, the point on the stress profile corresponding to  $r = a$  must be at C on the yield surface (fig.4). Point C ( $\sigma_c/2K = -1/2$ ,  $\sigma_r/2K = -1$ ) is independent of  $\zeta$  and has an associated strain rate *vector* with components  $\dot{\epsilon}_c = 0$  and  $\dot{\epsilon}_r = -\dot{\epsilon}_z$ .

Fig.6 is a plot of  $P^{1/2}K$  vs.  $\zeta$  for a pressurized ring in plane stress with  $b/a = 1.8$ , showing the lower bound, eq.(22), and the upper bounds, eq.(16) and  $P^{1/2}K = 1$ . For an isotropic material  $\zeta = 0$ , the limit pressure (fig. 6) is  $P^{1/2}K \approx 0.58$ . As the material is strengthened circumferentially,

the limit pressure increases ( $P^{1/2}K \approx 0.84$  at  $\zeta = 0.6$ ). The limit pressure continues to increase with increasing  $\zeta$  until a *critical value* is reached (here,  $\zeta \approx 0.8$ ) and the failure mode becomes one of local (*out of plane*) flow at  $r = a$ . Further circumferential strengthening is ineffective in increasing the limit pressure. As  $\zeta$  increases beyond the critical value, the failure mode remains one of local flow with  $P^{1/2}K = 1$ ; we refer to this condition as *over-reinforcement*. Analogous behavior is found by Lance and Robinson (1972) in other reinforced structures, e.g., beams, plates, etc. We note that the upper bound in fig.6 corresponding to eq.(16), i.e., the exact limit load under plane strain, continues to increase with  $\zeta$ , becoming infinite as  $\zeta \rightarrow 1$ . Thus, over-reinforcement in the sense discussed here does not occur under plane strain conditions.

The critical  $\zeta$  in plane stress for a given  $b/a$  is obtained from eq.(22) by setting  $P^{1/2}K = 1$ . This is illustrated in the plot of  $b/a$  vs.  $\zeta$  in fig.7. Alternatively, this graph can be interpreted as giving the *critical b/a ratio* at which the local (*out of plane*) failure mode takes effect for a given  $\zeta$ . For *thick* rings with  $b/a \geq$  the critical ratio, the ring fails locally at  $r = a$  and  $P^{1/2}K = 1$ .

Fig.8 shows the yield surface in plane stress, eq. (20), for  $\zeta = 0.8$ . The stress profile a-b in fig.8 (solid curve) corresponds to the plane stress solution discussed above for  $b/a = 1.8$ . Note that point a (i.e.,  $r = a$ ) on the stress profile coincides with point C (cf.,fig.4), which, as remarked earlier, is a necessary condition for the local (*out of plane*) failure mode. The dotted curve in fig.8 represents a stress profile typical for  $b/a < 1.8$ , i.e., for a *thinner* ring.

We shall apply the results of these limit pressure calculations to the NASA/PW test ring in the following sections. However, first we characterize the SiC/Ti material using available experimental data.

## CHARACTERIZATION OF SiC/Ti

The dotted curves in fig.9 represent longitudinal and transverse tensile data on SiC/Ti at 427 C. The solid curves are the perfect plasticity idealizations of these data used in the subsequent limit pressure calculations. The yield stress in the fiber direction  $Y_l$  is taken as being coincident with the fracture strength in that direction, viz.,  $Y_l = 200$  ksi. Two values are taken for the transverse yield strength,  $Y_t = 30$  ksi and  $Y_t = 35$  ksi delimiting a range of possible idealizations.

The material parameters  $\zeta$  and  $K$  corresponding to these choices of yield stress are obtained through eqs.(6) and are shown in the following table ( $Y_l$ ,  $Y_t$  and  $K$  are in ksi):

Table 1

$Y_l$	$Y_t$	$\zeta$	$K$
200	30	.983	15.04
200	35	.977	17.56

## APPLICATION TO THE NASA/PW TEST RING

The NASA/PW ring has a circumferentially reinforced SiC/Ti core covered with a layer of Ti cladding. Fig.10 shows the dimensions of the cross section and the relevant radii. Limit load calculations are made for two ring geometries ( $a$  and  $b$  are in inches), viz.,

Table 2

$b$	$a$	$b/a$
3.21	2.78	1.155
3.25	2.74	1.186

The first,  $b/a = 1.155$  corresponds to the actual radii of the SiC/Ti core. This calculation ignores the presence of the cladding, taking the strength of the ring to result entirely from the reinforced core. The second calculation,  $b/a = 1.186$  incorporates *effective* radii ( $b \approx 3.25$  and  $a \approx 2.74$ ) as a means of approximating the small contribution of strength of the Ti cladding. Here, the choice of radii amounts to enlarging the core dimensions by about one third of the cladding thickness. A more prudent choice must come from engineering experience based on experimental observation.

Critical values of  $\zeta$ , delimiting local failure in plane stress (fig.7), are  $\zeta = .992$  and  $.989$  for  $b/a = 1.155$  and  $1.186$ , respectively. As each of these values exceeds  $\zeta = .977$  and  $.983$ , identified in table 1 for the NASA/PW ring, we are assured that local failure is not a consideration here.

Plots of  $P^{1/2}K$  vs.  $\zeta$  for  $b/a = 1.155$  and  $1.186$  are shown in figs.(11) and (12), respectively. In each case, the dotted (upper) curve represents the exact limit pressure for plane strain and the upper bound for plane stress (eq.(16)). As discussed earlier, the upper bound  $P^{1/2}K = 1$  relating to local flow at  $r = a$  does not play a role here. The solid (lower) curve in each of the figs. (11) and (12) is the lower bound for plane stress (eq.(22)). Specific values of the limit pressure for plane strain  $P_s^1$  and the lower bound for plane stress  $P_e^1$  corresponding to  $\zeta$  and  $K$  in Table 1 and the ratios  $b/a$  in Table 2 are calculated as ( $K$ ,  $P_s^1$  and  $P_e^1$  are in ksi):

Table 3

$\zeta$	K	b/a	$P_s^1$	$P_e^1$
.983	15.04	1.155	<u>25.3</u>	28.8
.977	17.56	1.155	26.5	<u>29.0</u>
.983	15.04	1.186	<u>28.1</u>	34.2
.977	17.56	1.186	30.1	<u>34.4</u>

Table 3 indicates bounds on the limit pressure for  $b/a = 1.155$  as  $25.3 \text{ ksi} \leq P^l \leq 29.0 \text{ ksi}$ , spanning  $\pm 7\%$  about an average of  $27.15 \text{ ksi}$ . Recalling that  $b/a = 1.155$  implies the strength of the ring is attributable solely to the SiC/Ti core, we anticipate these bounds might tend to underestimate the strength of the NASA/PW ring.

Bounds for  $b/a = 1.186$  (in which limited strength of the cladding is accounted for through specifying *effective radii*) are  $28.1 \text{ ksi} \leq P^l \leq 34.4 \text{ ksi}$ . These range  $\pm 10\%$  about an average of  $31.25 \text{ ksi}$ . The lesser value  $28.1 \text{ ksi}$  corresponds to the lower bound for plane stress with a transverse yield strength of  $Y_t = 30 \text{ ksi}$  (fig.9); the greater value  $34.4 \text{ ksi}$  relates to plane strain with  $Y_t = 35 \text{ ksi}$ . The authors submit these bounds as their best estimate of the failure pressure of the NASA/PW benchmark ring at  $427\text{C}$ .

## CONCLUSIONS

The following accomplishments and conclusions are made in this study:

- The limit load is found for a circumferentially reinforced elastic-plastic ring (cylinder) subjected to interior pressure and in a condition of plane strain.
- Upper and lower bounds on the limit load are found for a reinforced ring (disc) under conditions of plane stress.
- These closed form solutions apply for all values of the material parameters  $0 \leq \zeta \leq 1$  and  $K$  (designating the degree of reinforcement and the yield stress in transverse shear, respectively), and any  $b/a$  (ratio of outer to inner radius). The results provide simple and useful tools for the engineer designing fiber reinforced metallic rings.

- As  $\zeta \rightarrow 1$  the limit pressure of the circumferentially reinforced ring in plane strain increases indefinitely. Conversely, the limit pressure of the ring in plane stress is bounded by its transverse ( $\approx$  matrix) strength, ultimately failing by local plastic (*out of plane*) flow directly under the applied pressure.

- The limit load calculations are applied to the NASA/PW SiC/Ti test ring giving an estimated range of the failure pressure at 427 C of  $\approx 28 - 34$  ksi.

- The calculated bounds on the limit pressure are useful in providing a check on more comprehensive finite element analyses.

## ACKNOWLEDGEMENT

Prof. D.N. Robinson acknowledges the support of this research through NASA grant NAG-3-379.

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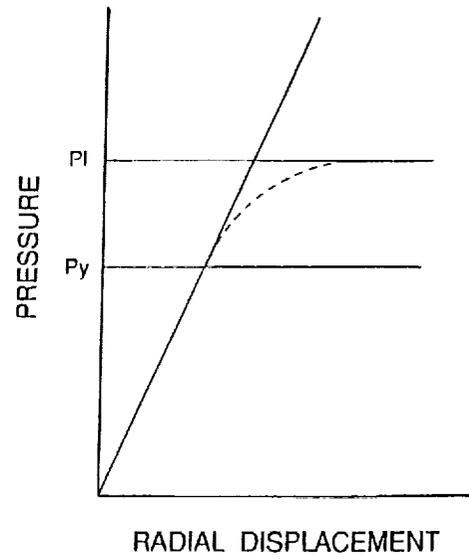


Fig.1 Schematic diagram of pressure vs. radial displacement.

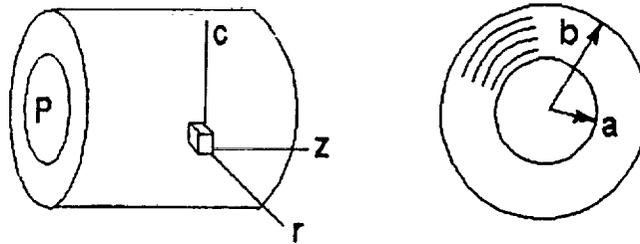


Fig.2 Cylindrical geometry ( $z, c, r$ ) of circumferentially reinforced ring.  $b =$  outer radius,  $a =$  inner radius.

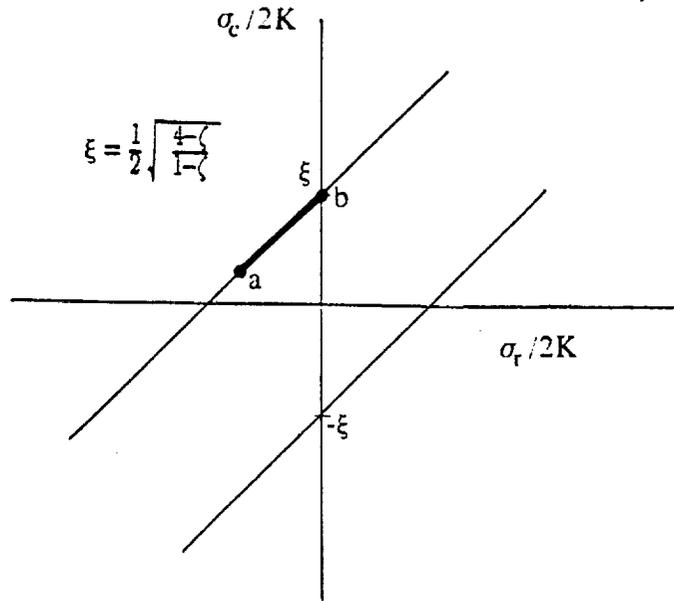


Fig.3 Yield surface for plane strain in the dimensionless stress space  $\sigma_r/2K, \sigma_c/2K$ . Typical stress profile a-b ( $r=a \rightarrow r=b$ ) for pressure loading.

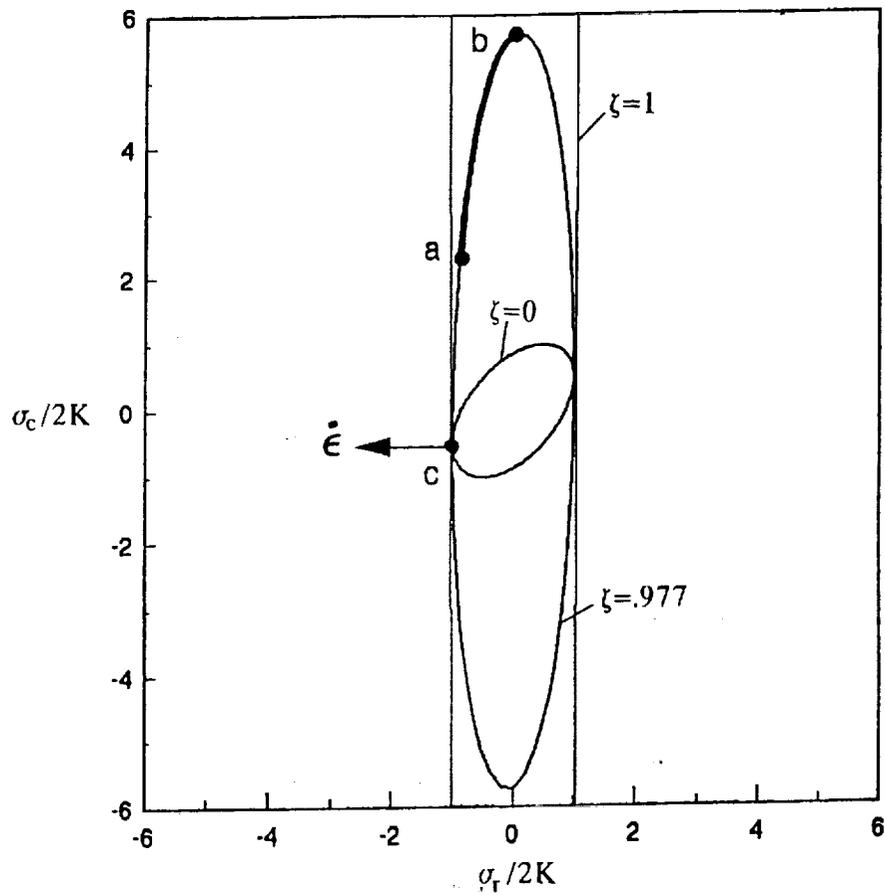
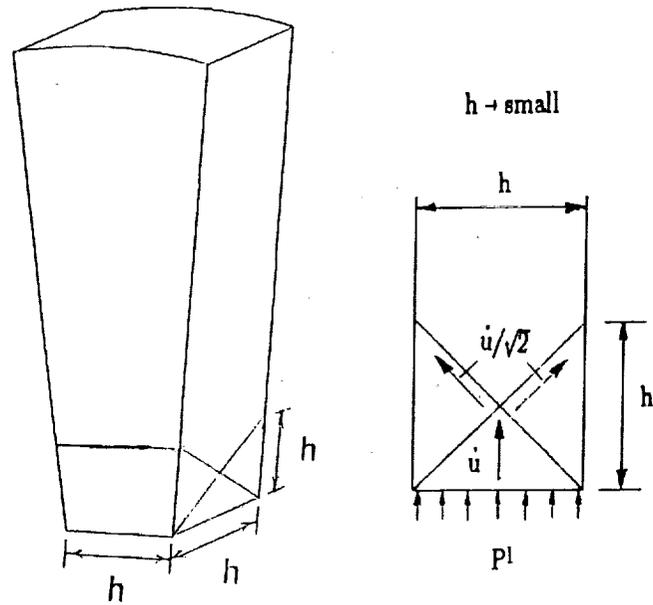


Fig.4 Yield surfaces for plane stress in  $\sigma_r/2K, \sigma_c/2K$  space. Surfaces are shown for  $\zeta = 0, \zeta = .977$  and  $\zeta = 1$ . Typical stress profile a-b for pressure loading with  $\zeta = .977$ .



$$P^l h^2 \dot{u} = 4[(\dot{u}/\sqrt{2})(\sqrt{2}h/2)(h)(K)]$$

$$P^l/2K = 1$$

Fig.5 Kinematically admissible slip line field for local (out of plane) flow at  $r = a$ . Upper bound calculation giving  $P^l/2K = 1$ .

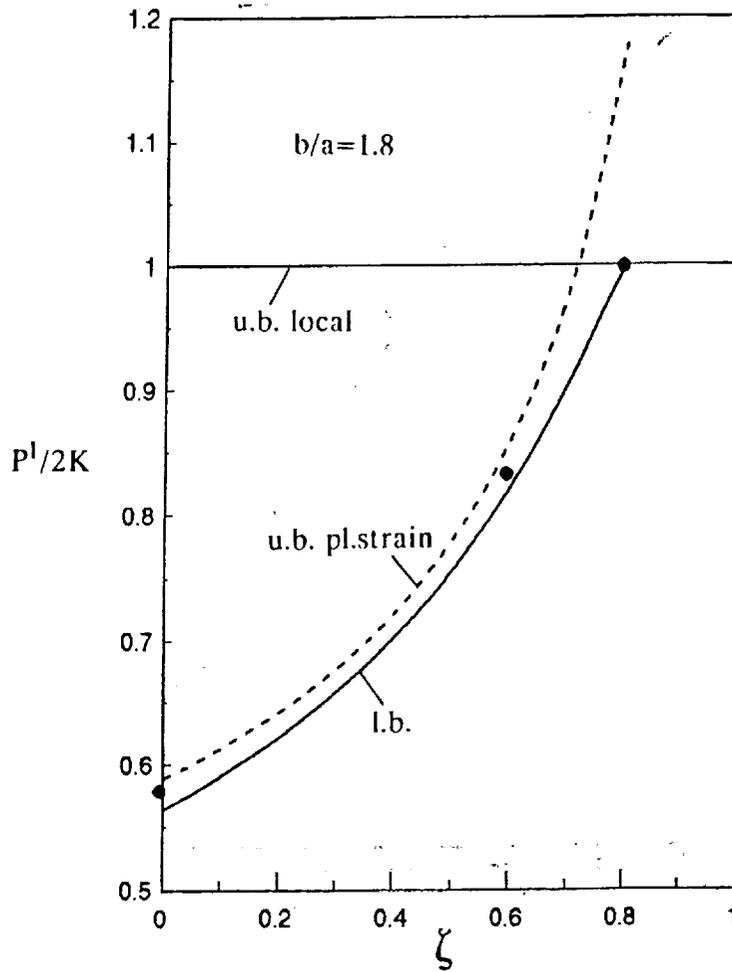


Fig.6  $P^{1/2}K$  vs.  $\zeta$  for a pressurized ring in plane stress with  $b/a = 1.8$ . Lower bound, eq.(22), and upper bounds, eq.(16) and  $P^{1/2}K = 1$ . Critical  $\zeta \approx 0.8$ .

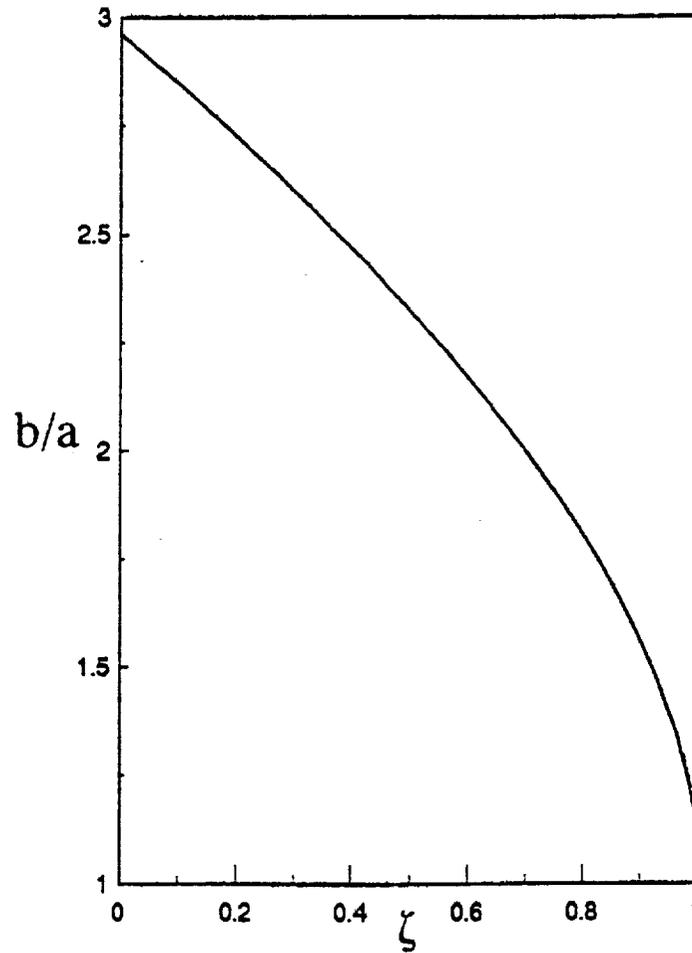


Fig.7  $b/a$  vs.  $\zeta$  for local (out of plane) failure in plane stress.  $P^{1/2}K = 1$ .

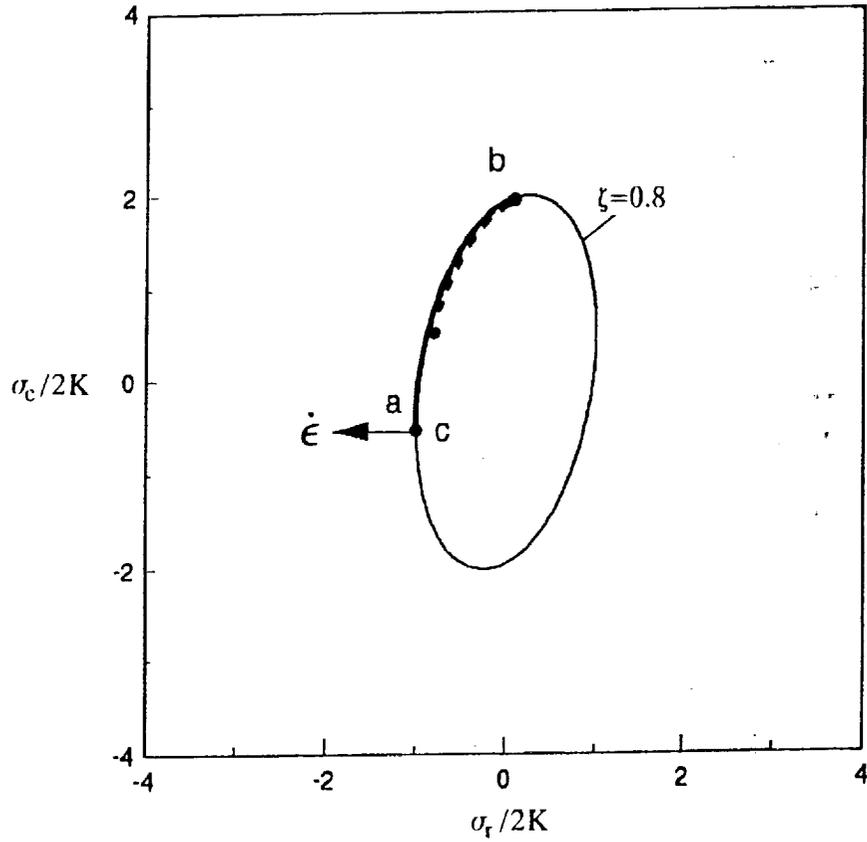


Fig.8 Yield surface in plane stress with  $\zeta = 0.8$ . Stress profile a-b (solid curve) for  $b/a = 1.8$ . Typical stress profile (dotted) for  $b/a < 1.8$ .

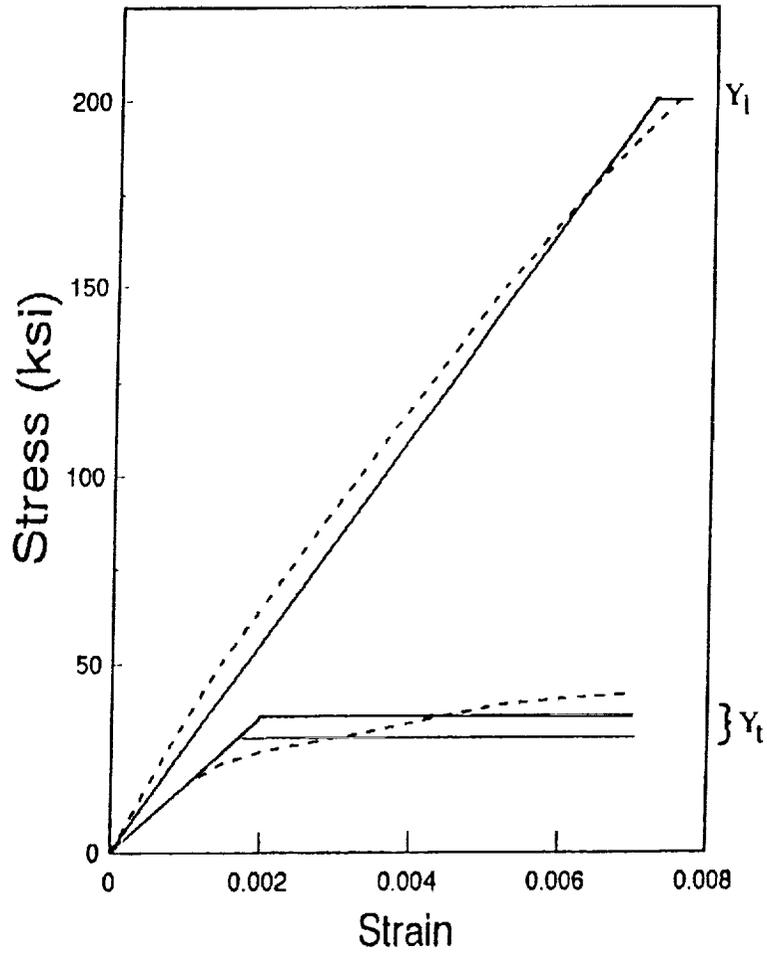


Fig.9 Longitudinal and transverse tensile curves (dotted) for SiC/Ti at 427 C. Elastic perfectly plastic idealizations (solid curves) indicating  $Y_l$  and  $Y_t$ .

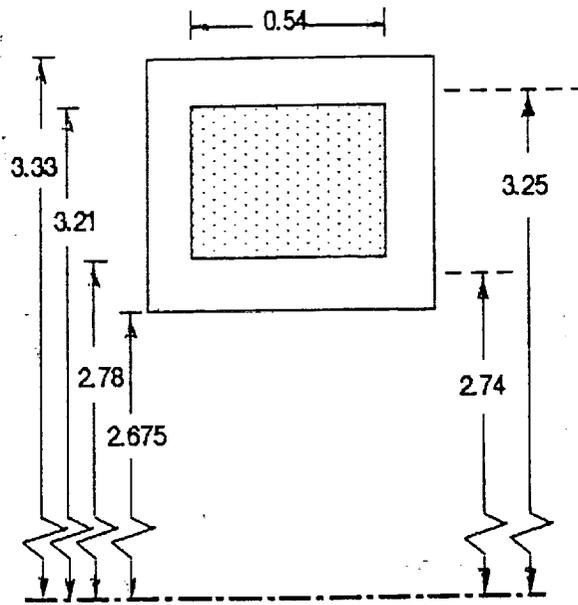


Fig.10 Cross section of NASA/PW test ring. Actual and *effective* radii.

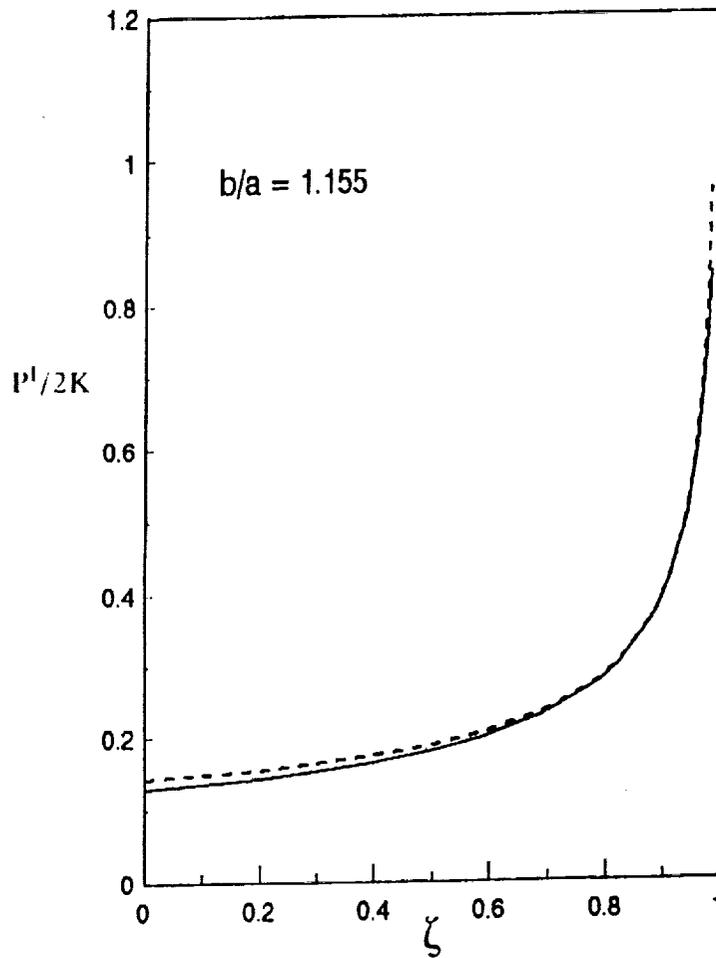


Fig.11  $P^l/2K$  vs.  $\zeta$  for  $b/a = 1.155$ . Plane strain limit pressure and plane stress upper bound (dotted). Plane stress lower bound (solid).

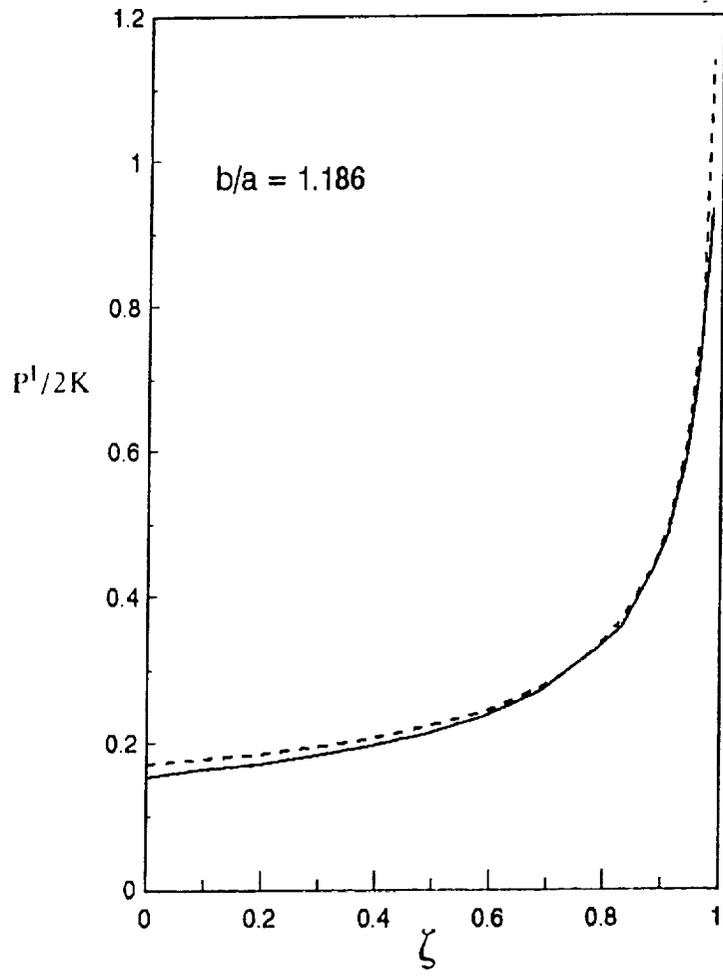


Fig.12  $P^{1/2}K$  vs.  $\zeta$  for  $b/a = 1.186$ . Plane strain limit pressure and plane stress upper bound (dotted). Plane stress lower bound (solid).

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<b>1. AGENCY USE ONLY</b> (Leave blank)	<b>2. REPORT DATE</b> October 1991	<b>3. REPORT TYPE AND DATES COVERED</b> Final Contractor Report	
<b>4. TITLE AND SUBTITLE</b> Limit Pressure of a Circumferentially Reinforced SiC/Ti Ring		<b>5. FUNDING NUMBERS</b>  WU-510-01-50 G-NAG3-379	
<b>6. AUTHOR(S)</b> D.N. Robinson and M.S. Pastor			
<b>7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)</b>  University of Akron Department of Civil Engineering Akron, Ohio 44325 - 3905		<b>8. PERFORMING ORGANIZATION REPORT NUMBER</b>  None	
<b>9. SPONSORING/MONITORING AGENCY NAMES(S) AND ADDRESS(ES)</b>  National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135 - 3191		<b>10. SPONSORING/MONITORING AGENCY REPORT NUMBER</b>  NASA CR - 187211	
<b>11. SUPPLEMENTARY NOTES</b> Project Manager, Steven M. Arnold, Structures Division, NASA Lewis Research Center, (216) 433 - 3334.			
<b>12a. DISTRIBUTION/AVAILABILITY STATEMENT</b>  Unclassified - Unlimited Subject Category 39		<b>12b. DISTRIBUTION CODE</b>	
<b>13. ABSTRACT</b> (Maximum 200 words)  Limit loads under plane stress and plane strain are found for a circumferentially reinforced elastic-plastic ring subjected to interior pressure. These are used as bounds on an estimate of the failure pressure of a SiC/Ti test rig that is being fabricated and tested under the co-sponsorship of NASA Lewis Research Center and Pratt and Whitney Aircraft. The ring is to serve as a benchmark against which deformation and failure analysis methods can be assessed. An anisotropic perfect plasticity idealization of the SiC/Ti ring material is made and used in the limit load calculations. An estimate of the failure pressure of the NASA/PW benchmark test ring is given.			
<b>14. SUBJECT TERMS</b> Limit load; Anisotropic; Perfect plasticity; Metal matrix composite; Continuum mechanics		<b>15. NUMBER OF PAGES</b> 26	
		<b>16. PRICE CODE</b> A03	
<b>17. SECURITY CLASSIFICATION OF REPORT</b> Unclassified	<b>18. SECURITY CLASSIFICATION OF THIS PAGE</b> Unclassified	<b>19. SECURITY CLASSIFICATION OF ABSTRACT</b> Unclassified	<b>20. LIMITATION OF ABSTRACT</b>